Homework 6 (due Mon. Apr. 30, 2012)
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This homework assignment covers selected topics in Chapters 7, 8, and 9.

Problem 1

Express the formula for direct CP violation asymmetry for a particle P decay to a final state f using only real numbers:

$$\mathcal{A}_{CP} \equiv (\Gamma(P \to f) - \Gamma(\bar{P} \to \bar{f}))/(\Gamma(P \to f) + \Gamma(\bar{P} \to \bar{f}))$$

Assume that the decay mechanism is dominated by two decay amplitudes with different weak (ϕ_1, ϕ_2) and strong (δ_1, δ_2) interaction phases.

Hint: remember that the weak phase changes sign for the conjugate process, while the strong phase does not:

$$A(P \to f) = A_1 e^{i\phi_1 + i\delta_1} + A_2 e^{i\phi_2 + i\delta_2}$$
$$A(\bar{P} \to \bar{f}) = A_1 e^{-i\phi_1 + i\delta_1} + A_2 e^{-i\phi_2 + i\delta_2}$$

Show that the result is proportional to sine of the phase differences.

Problem 2

In class we discussed the decay $K^0 \to \mu^+ \mu^-$ proceeding through the box diagram with two virtual W bosons. We showed that introduction of the c quark as the second quark in the box resolved the puzzle of the decay suppression (assuming only two families of quarks). Show that this suppression still holds with three quark families (with t quark in the box).

Note: this mechanism is fully effective when the masses of the internal quarks are the same $m_u = m_c = m_t$. In reality $m_t > m_W \gg m_u, m_c$ and this decay is possible but highly suppressed.

Show that $K^0 \to e^+ e^-$ has higher helicity suppression relative to $K^0 \to \mu^+ \mu^-$.

Hint: the two-quark mixing with the Cabibbo angle is replaced by the unitary CKM matrix with three quarks.

Problem 3

The D^+ meson can decay to a number of various final states. Without doing detailed calculations of the matrix elements and phase-space, what ratio of the decay rates do you expect for the following channels:

(a) $D^+ \to K^- \pi^+ \pi^+$
(b) $D^+ \to K^+ \pi^+ \pi^-$
(c) $D^+ \to \pi^+ \pi^+ \pi^-$

Hint: draw Feynman diagrams in each case.

Problem 4

Suppose that we could have a T^+ meson ($t\bar{d}$). What is the most likely sequence of decay down to stable particles? You can leave pions and leptons without considering their further decay.

Problem 5 (similar to Problem 9.3 from the textbook)

In an experiment using a reactor as a source, the observed rate of $\bar{\nu}_e$ reactions at a distance of 250 m from the reactor core is found to be 0.95 ± 0.10 of that expected. If the mean effective anti-neutrino energy is 5 MeV, what limits would this place on a possible neutrino mass difference, assuming a mixing angle $\theta = 45°$?

Compute the answer for two confidence levels of the limit: 2.28% and 0.14%. Assume that the error corresponds to the Gaussian distribution and represents 1σ uncertainty. This will translate into 2σ and 3σ deviations for the above two confidence levels.

Problem 6

(a) The photon does not couple to particles without charge, such as Z^0 boson. However, one can draw a Feynman diagram which would allow Z^0 transition to two photons. Draw an example of such a diagram. Note that such a decay is actually forbidden by the Landau-Yang theorem. Having two massless photons in a spin-1 state would violate Bose-Einstein statistics for two identical bosons. Nonetheless, a search for the decay $Z^0 \to \gamma\gamma$ has been performed experimentally and the limit was set at $< 5.2 \times 10^{-5}$.

(b) In the Standard Model, elementary particles obtain mass by coupling to a scalar field. The existence of a Higgs boson is a consequence of such a theory. This part of the theory has not been confirmed yet experimentally with high enough confidence. In order to detect a Higgs boson, scientists expect to reconstruct its decay products, such as $H \to Z^0 Z^0$ for example (where the Z^0 bosons may also appear off-shell). Suggest the final states which could be detected in the experiment and provide clean observation of a heavier Higgs.

The Higgs decay to two photons is also a promising channel for the Higgs discovery. However, Higgs does not couple to massless particles. Draw a Feynman diagram to explain how this decay could happen.