Space-Time

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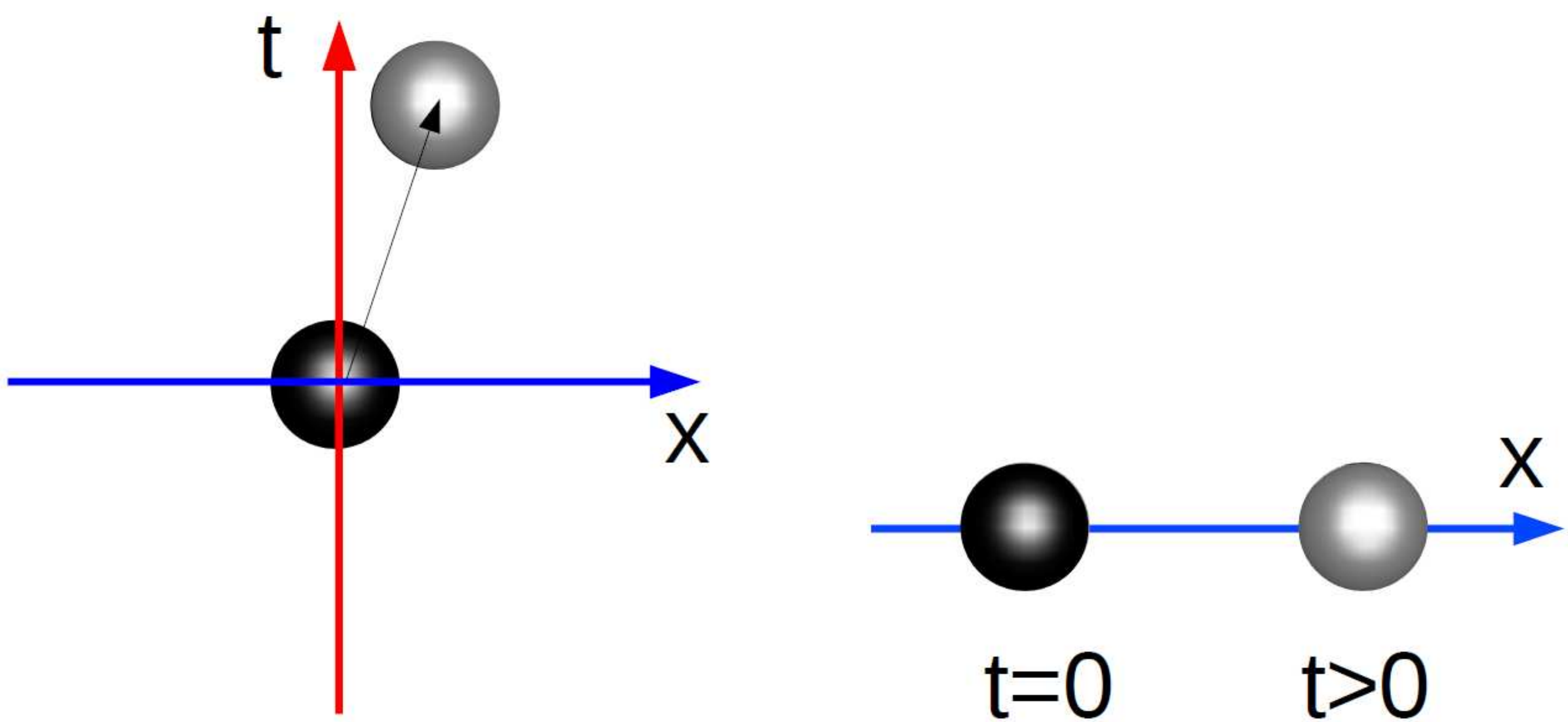
What is Space-Time?

- Space-time: combination of space and time

  3 dimensions of space (though this is being questioned)

  time is the 4th dimension

- It is a mathematical model to write physics laws



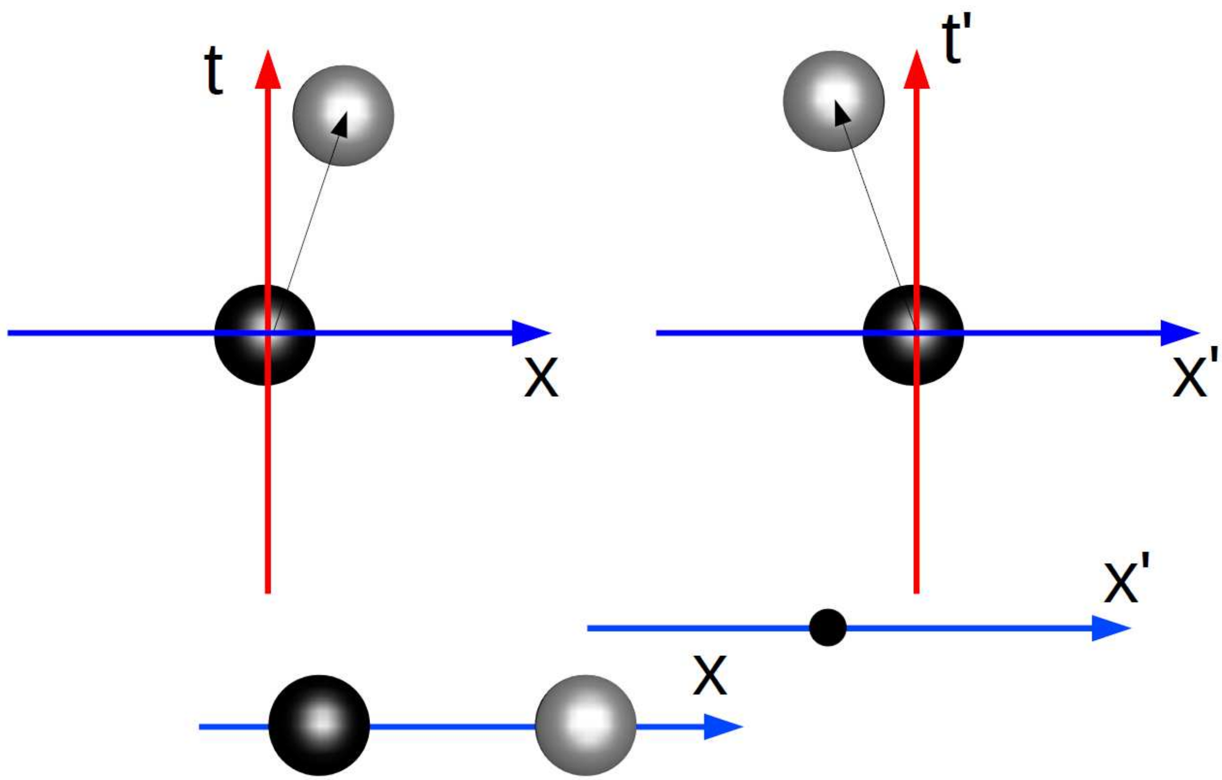
Space-Time

- Space-time in classical physics (Galilean transformation)

  $\Rightarrow$ time is absolute and independent of space

- In Special Relativity (and later General)

  $\Rightarrow$ time and space are related $\Rightarrow$ time different in frames



Problem of Classical Physics (1900's)

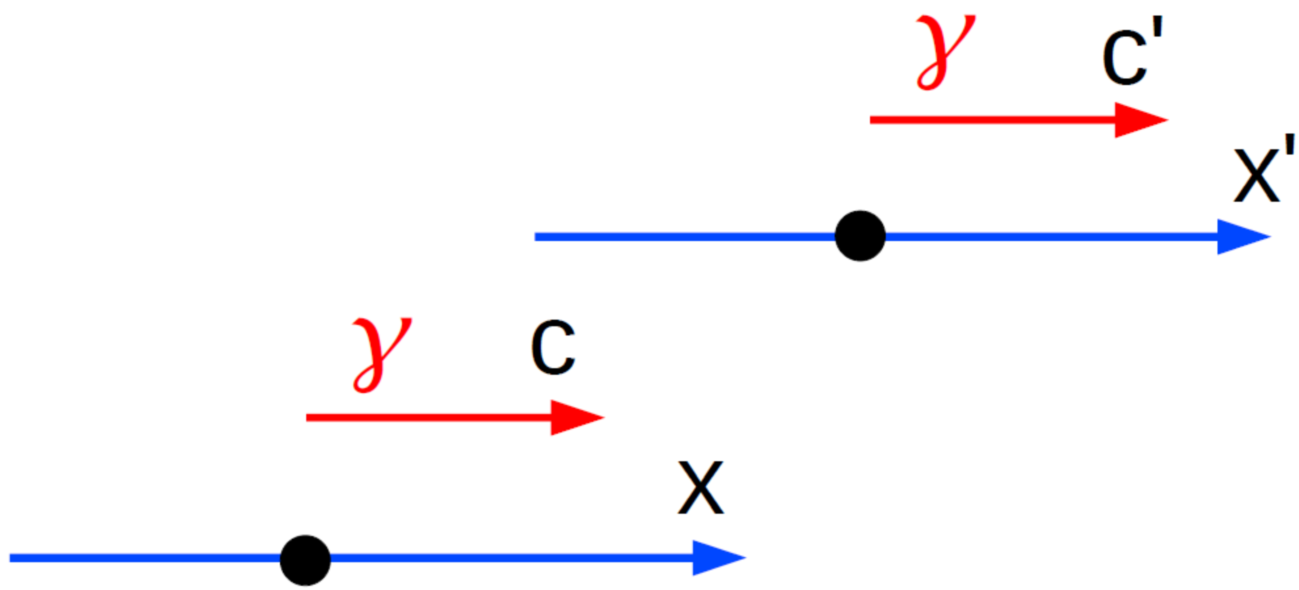
- Relativity and E&M looked inconsistent

  relativity – same physics laws in all systems

  E&M – Maxwell's equations involve speed of light $= c$

  Classically: speed of light different in different systems

  classically $c' = c - v$



Postulate of Special Relativity

- Einstein in 1905:
  - relativity valid – same physics laws in all systems
  - speed of light – same in all systems
- What is speed of light?

$$c = 299\,792\,458 \text{ m/s} \simeq 3 \times 10^8 \text{ m/s}$$

- Breaks classical Newtonian mechanics

$$t = t' \tag{1}$$

$$x = x' + v \times t' \tag{2}$$

$$y = y' \tag{3}$$

$$z = z' \tag{4}$$

Consequence of Einstein's postulates

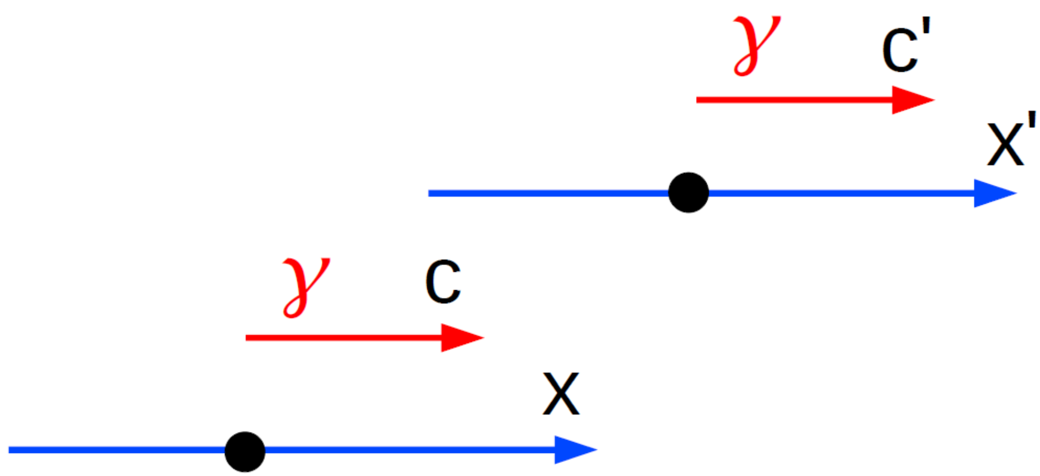
- Special Relativity

$$t = \left(t' + \frac{v}{c^2} \times x'\right) / \sqrt{1 - \frac{v^2}{c^2}} \tag{5}$$

$$x = \left(x' + v \times t'\right) / \sqrt{1 - \frac{v^2}{c^2}} \tag{6}$$

$$y = y' \tag{7}$$

$$z = z' \tag{8}$$



Did the trick work?

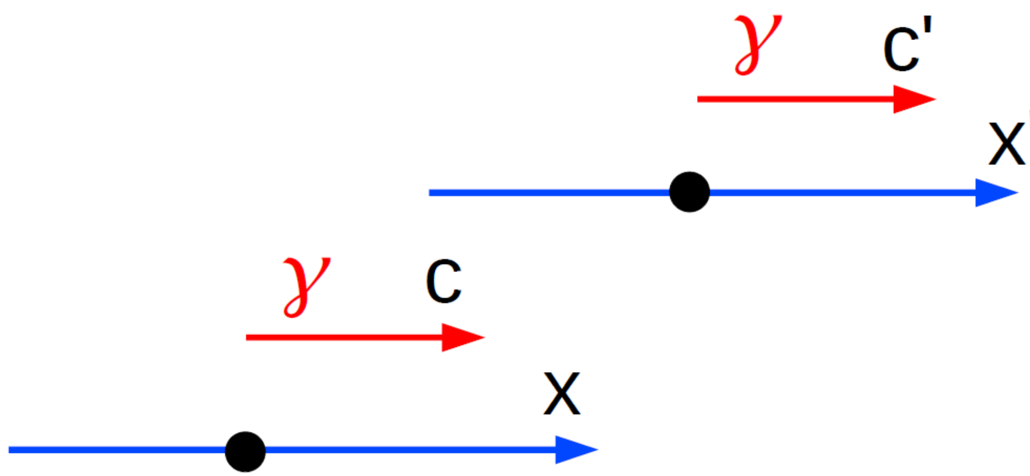
- A photon (light) will travel $x = c \times t$

$$x = (x' + v \times t') / \sqrt{1 - \frac{v^2}{c^2}} = c \times t$$

$$t = (t' + \frac{v}{c^2} \times x') / \sqrt{1 - \frac{v^2}{c^2}}$$

$\Rightarrow (x' + v \times t') = (ct' + \frac{v}{c} \times x')$

$\Rightarrow c' = \frac{x'}{t'} = \frac{c - v}{1 - v/c} \equiv c$ $\Rightarrow$ same speed of light



Consequences

- Length contraction

  When a body moves with speed v relative to the observer, its length is contracted in the direction of motion by $\times\sqrt{1-\frac{v^2}{c^2}}$

- Time dilation

  When a clock moves with speed v relative to the observer, its rate is measured to have slowed down by $\times\sqrt{1-\frac{v^2}{c^2}}$

Length contraction

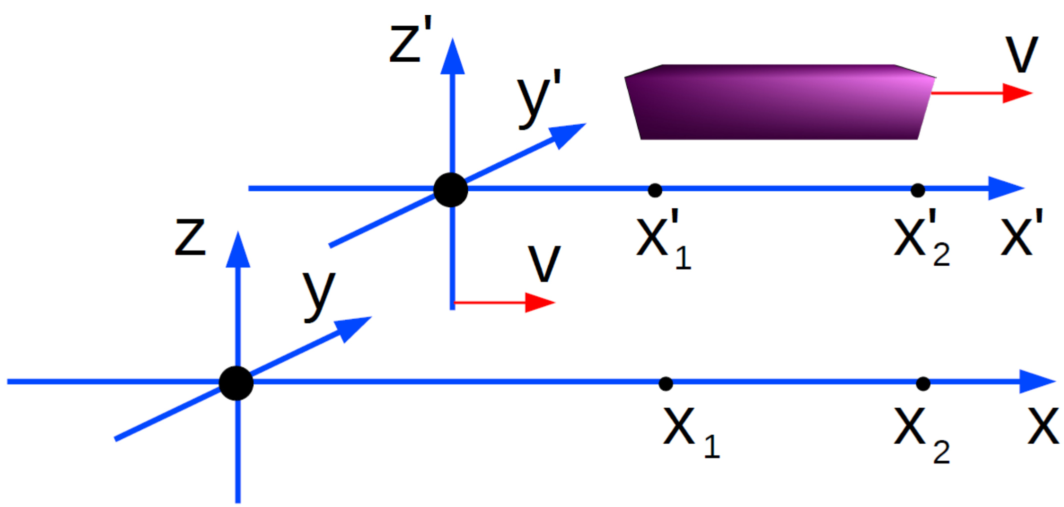
- Consider a moving rod of length L in our frame

$$L = (x_2 - x_1) \text{ at } t_1 = t_2$$

- Length in its own frame $L'_0 = (x'_2 - x'_1) > L$, proof:

$$L'_0 = (x_2 - v \times t_2)/\sqrt{1 - \frac{v^2}{c^2}} - (x_1 - v \times t_1)/\sqrt{1 - \frac{v^2}{c^2}}$$

$$= L/\sqrt{1 - \frac{v^2}{c^2}} > L \quad \Rightarrow \quad L = L'_0 \times \sqrt{1 - \frac{v^2}{c^2}}$$



Time dilation

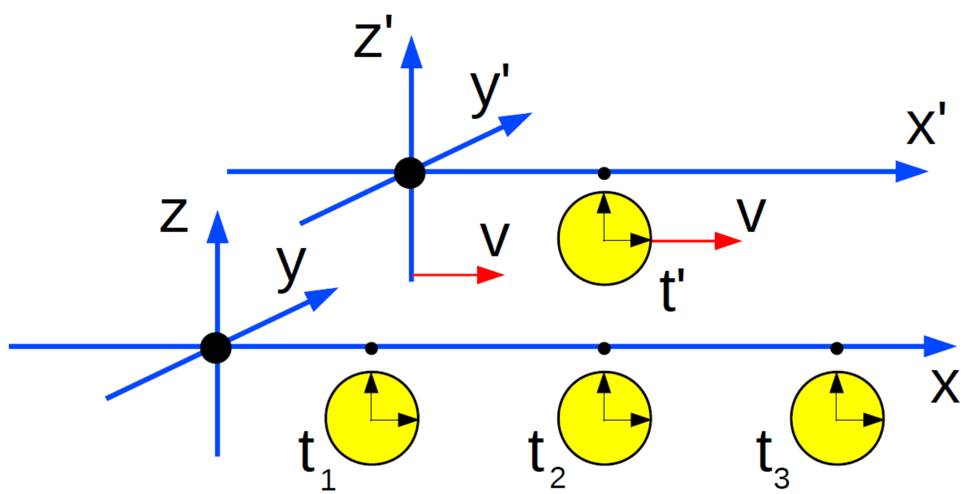
- Consider moving clock at x' =const in its own frame

$$\Delta t'_0 = (t'_2 - t'_1)$$

- Time between two events in our frame $\Delta t = (t_2 - t_1)$

$$\Delta t = (t'_2 + \frac{v}{c^2}x')/\sqrt{1-\frac{v^2}{c^2}} - (t'_1 + \frac{v}{c^2}x')/\sqrt{1-\frac{v^2}{c^2}}$$

$$\Delta t = \Delta t'_0/\sqrt{1-\frac{v^2}{c^2}}$$



Length contraction and time dilation

- Length contraction

  When a body moves with speed v relative to the observer, its length is contracted in the direction of motion by $\times\sqrt{1-\frac{v^2}{c^2}}$

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Time dilation in particle physics

- Example: lets take cosmic ray muon μ^+

  some facts: every minute 1 muon goes through 1 cm^2 area

  Quarknet: comsic ray detectors, muon lifetime experiment

- Lets take typical muon energy 3 GeV

  – speed $= 0.9994 \times c$

  – lifetime $\Delta t'_0 = 2.2 \times 10^{-6}$ seconds

  – naive distance traveled (if there were no time dilation)

  $$= 0.9994 \times c \times \Delta t'_0 = 659 \text{ m}$$

  – but $1/\sqrt{1 - \frac{v^2}{c^2}} = 29$

  – distance travel $\Delta L = 659\text{m} \times 29 = 19\,000$ m $= 19$ km

Time dilation: muon

- Muon with $v = 0.9994 \times c$ and $\gamma = 1/\sqrt{1 - \frac{v^2}{c^2}} = 29$

- From our point of view

muon is moving speed $v \simeq c$

time dilated

$$\Delta t = \Delta t'_0 \times \gamma$$

– distance $\Delta L = 19$ km

– time $\Delta t = 64 \times 10^{-6}$ s

- From muon point of view

Earth is moving speed $v \simeq c$

distance contracted

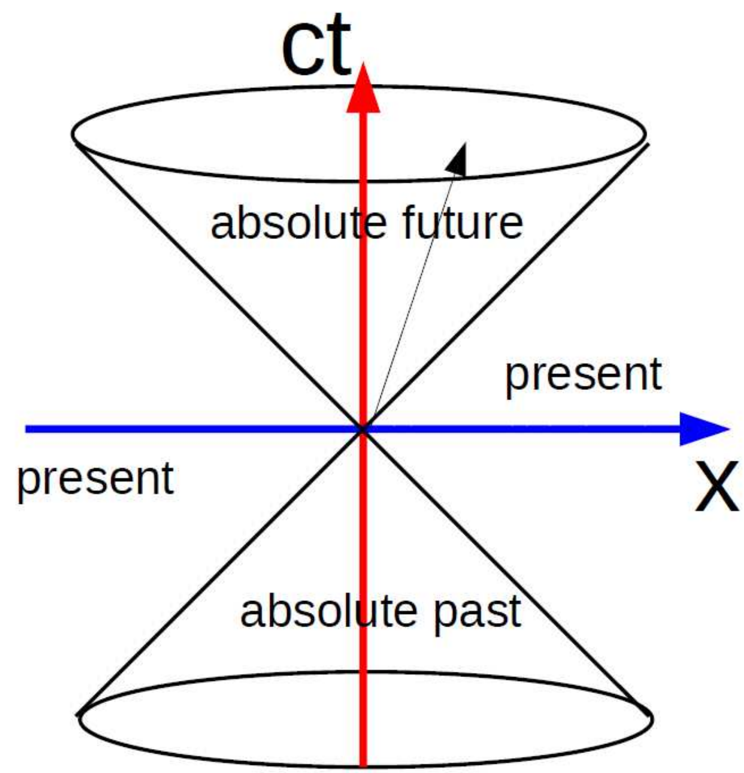
$$\Delta L'_0 = \Delta L/\gamma$$

$$\Delta L'_0 = 659 \text{ m}$$

$$\Delta t'_0 = 2.2 \times 10^{-6} \text{ s}$$

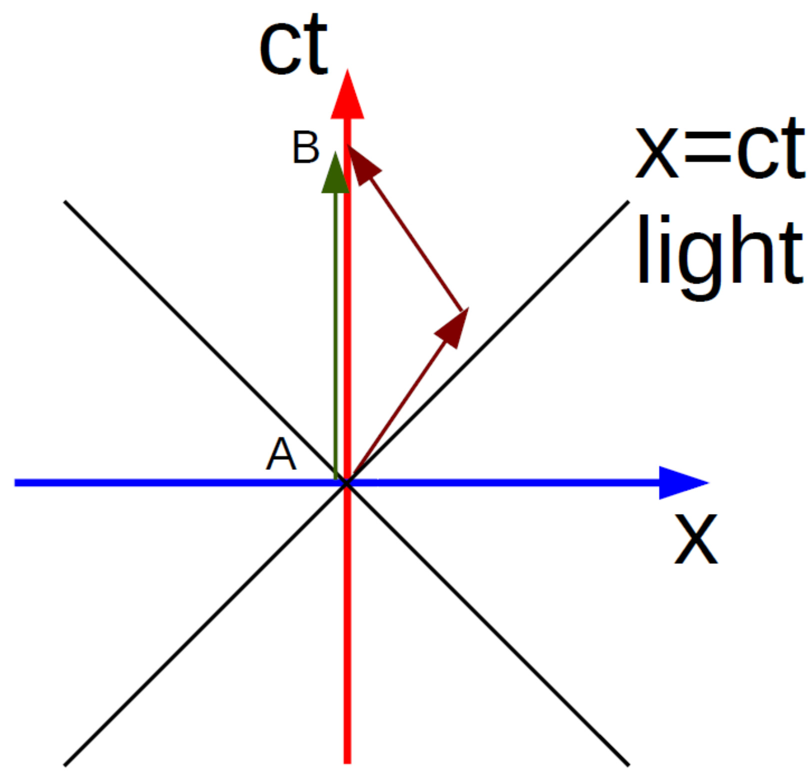
“Time Travel”

- Cannot move “backwards in time”
- Move “forward” with different speed
- You can change “your clock”, but not direction:
  - biology: hybernate
  - physics: relativistic speed



Twin paradox

- If one could reach $v \simeq 0.9995 \times c$ (we will see why not)
  - time $\gamma = 1/\sqrt{1-\frac{v^2}{c^2}} \simeq 30$ times slower (like muon earlier)
  - get on a rocket and come back in 90 years
  - twin on Earth 90 years older
  - twin on rocket 3 years older



Conservation of Energy and Momentum?

- Energy and Momentum conserve, but different definition:

$$E = mc^2 = m_0c^2 \Big/ \sqrt{1 - \frac{v^2}{c^2}}$$

$$p = mv = m_0v \Big/ \sqrt{1 - \frac{v^2}{c^2}}$$

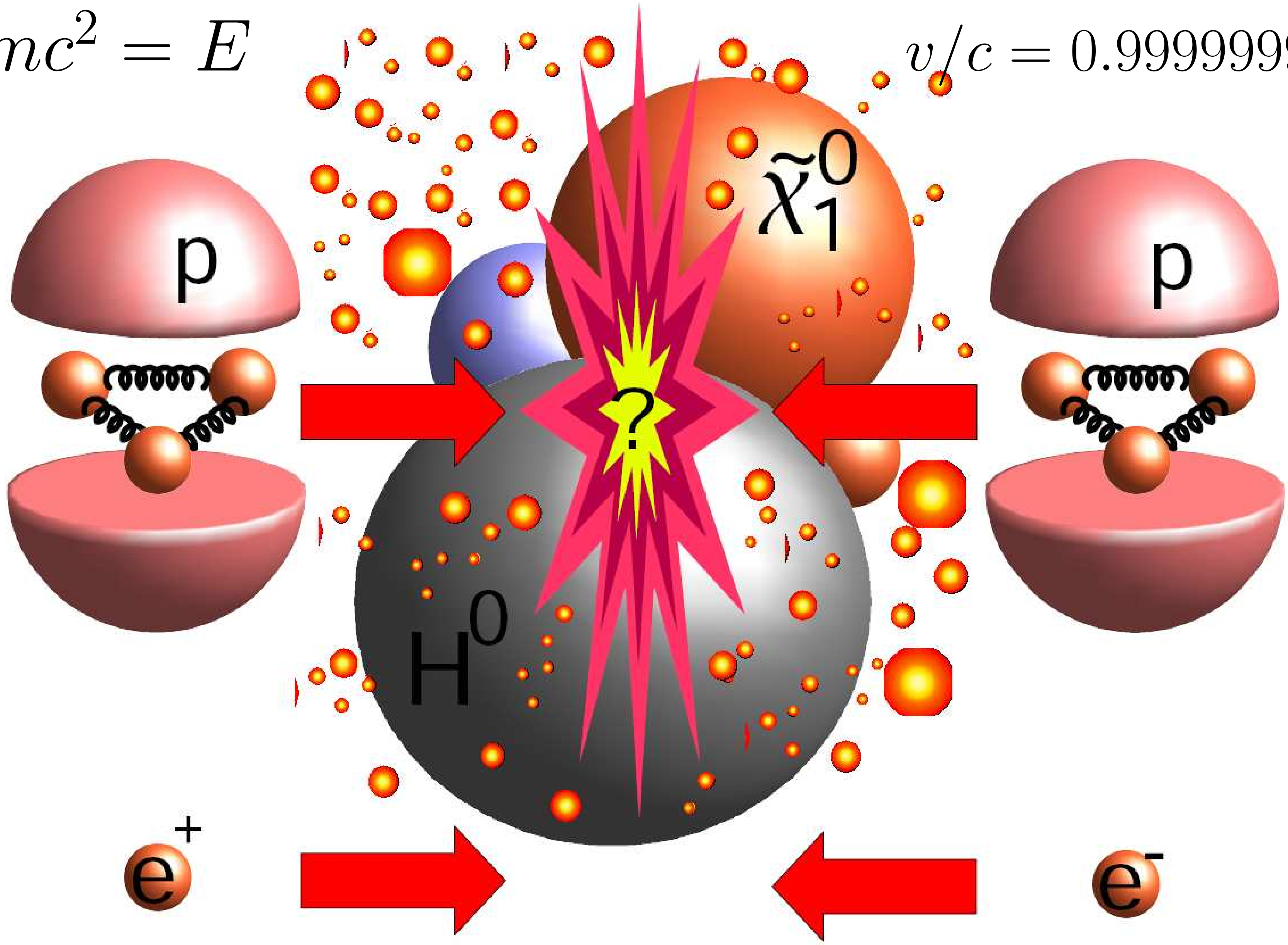
- Impossible to reach speed of light $v = c \;\Leftrightarrow\; E = \infty$
  unless you are massless ($m_0 = 0$, like photon)

- Reverse is true: massless $\Rightarrow\; v = c$ in all frames
  otherwise $p = 0$ and $E = 0 \Rightarrow$ like nothing

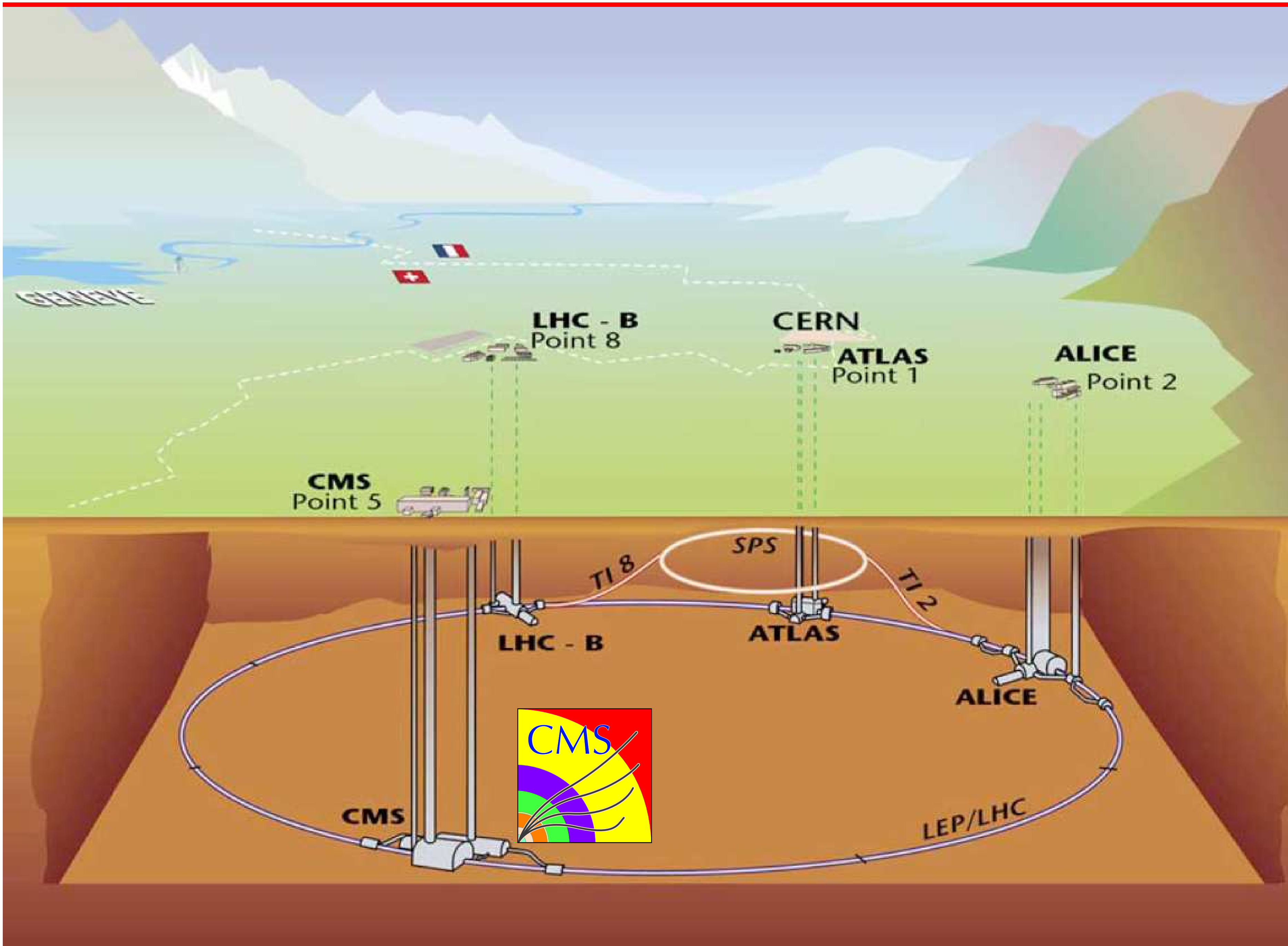
- One can convert mass into energy and energy into mass
  relativistic mass is energy

Reaching Highest Energy

- $mc^2 = E$ $\qquad v/c = 0.999999991$



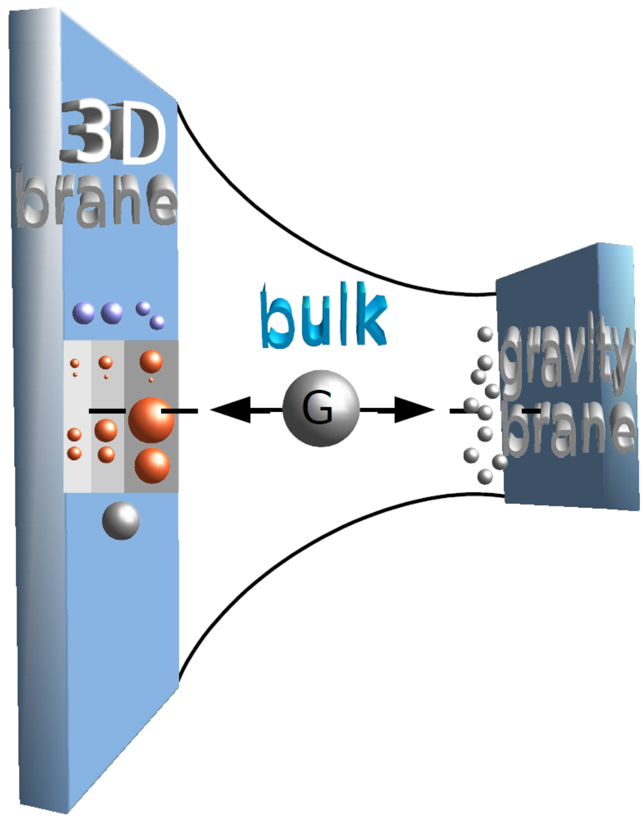
Large Hadron Collider: in Operation Now



Extra Dimensions of Space?

- Ideas of extra dimensions of space-time

  limit size of 5th dim $d_5 < 10^{-19}$ m



- Extra dimensions, Higgs, and other searching on LHC

What Gives Mass to Baryons: Proton, Neutron,...

- Remember Einstein's formula

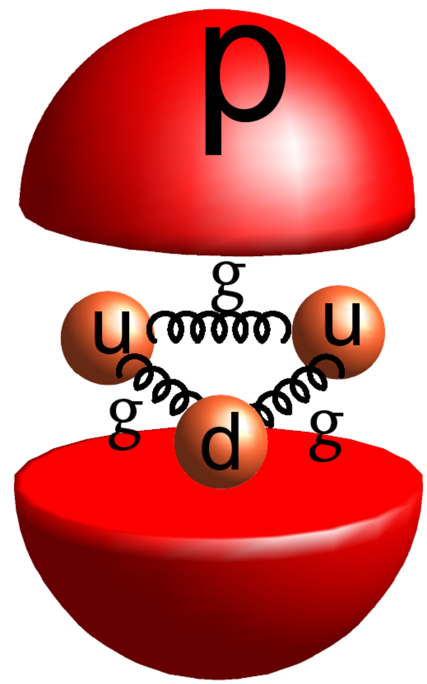
$$E = mc^2$$

m(u or d) $< 1\%$ m(proton)

Mostly energy of gluons and quarks inside gives proton mass

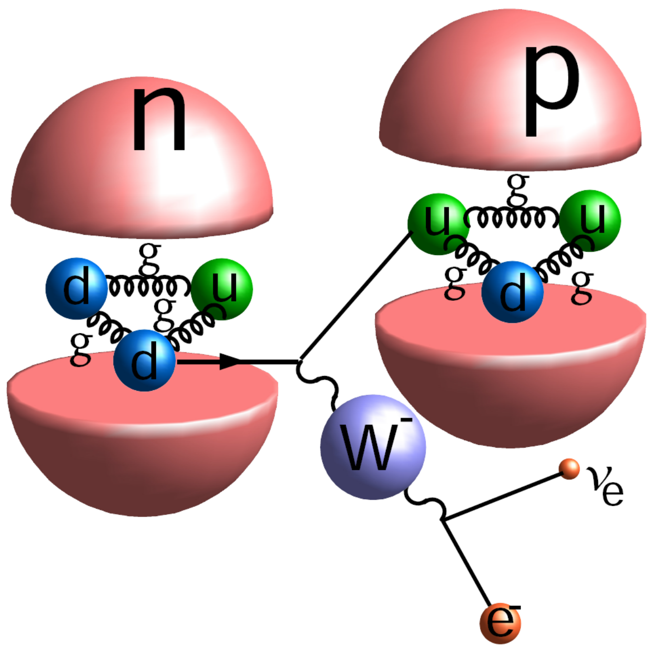
not Higgs mechanism directly

but it is important ($m_d > m_u$)



But Higgs Mechanism is Very Important

- Makes Weak Interactions weak: mass of Z, W^-, W^+



similarly first step in sun fusion

$$p + p \to d + e^+ + \nu_e$$

- Makes certain hierarchy of masses

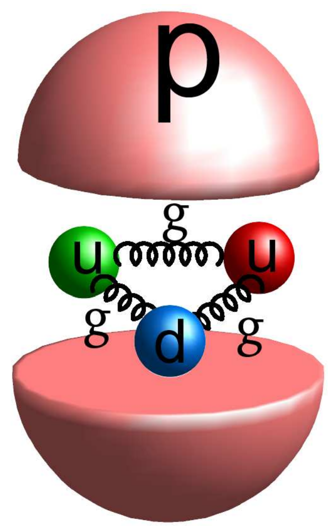
  essential for our existence

Hypothetical Scenario: Different Quark Mass

- Again, normally proton is stable and neutron decays:

  $\mathsf{m}(n) > \mathsf{m}(p) + \mathsf{m}(e) + \mathsf{m}(\nu_e)$

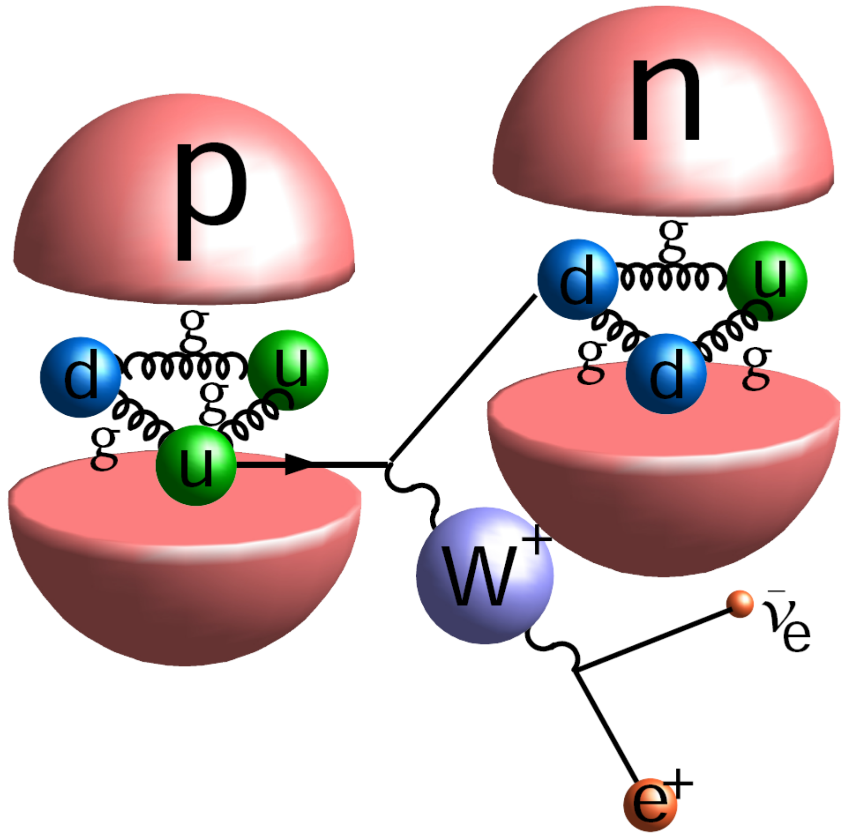
- Why is $\mathsf{m}(n) > \mathsf{m}(p)$
  - $\mathsf{m}(p) = 938$ MeV, $\mathsf{m}(n)$ – $\mathsf{m}(p) = 1.3$ MeV
  - tiny difference makes a big difference!
  - naively expect $\mathsf{m}(p) > \mathsf{m}(n)$ if u and d were the same
  - but $\mathsf{m}(d) > \mathsf{m}(u)$



- New scenario:
  - what if $\mathsf{m}(d) < \mathsf{m}(u)$

Hypothetical Scenario: Different Quark Mass

- If m(d) $<$ m(u), proton decays:



- Consequence: no Hydrogen, no $H_2 0$, no life
  – still have He^4, rapid nn fusion, instead of slower pp