

# Space-Time

Andrei Gritsan

Johns Hopkins University



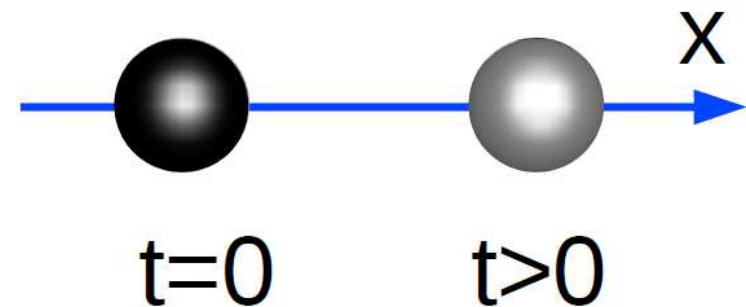
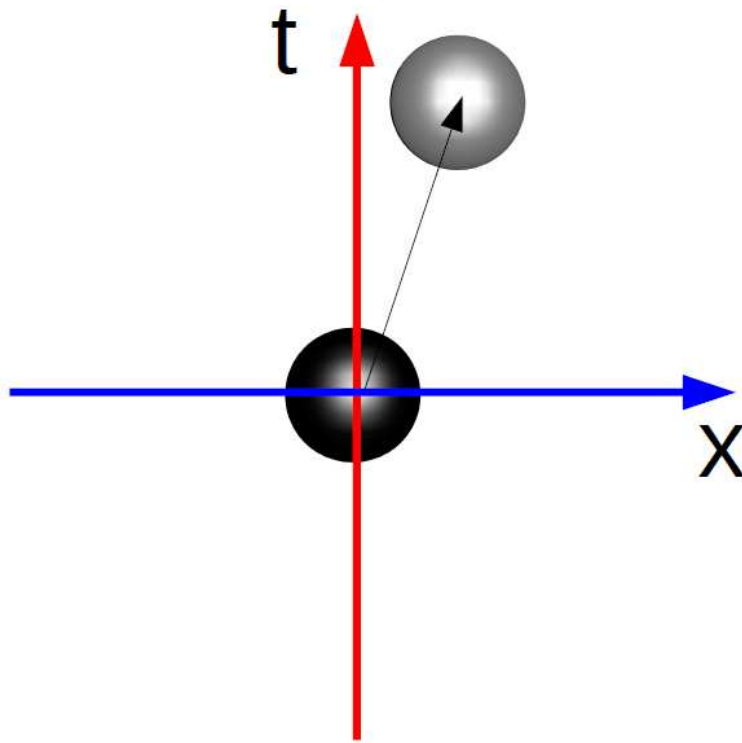
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Special Relativity 171.201/171.207

# What is Space-Time?

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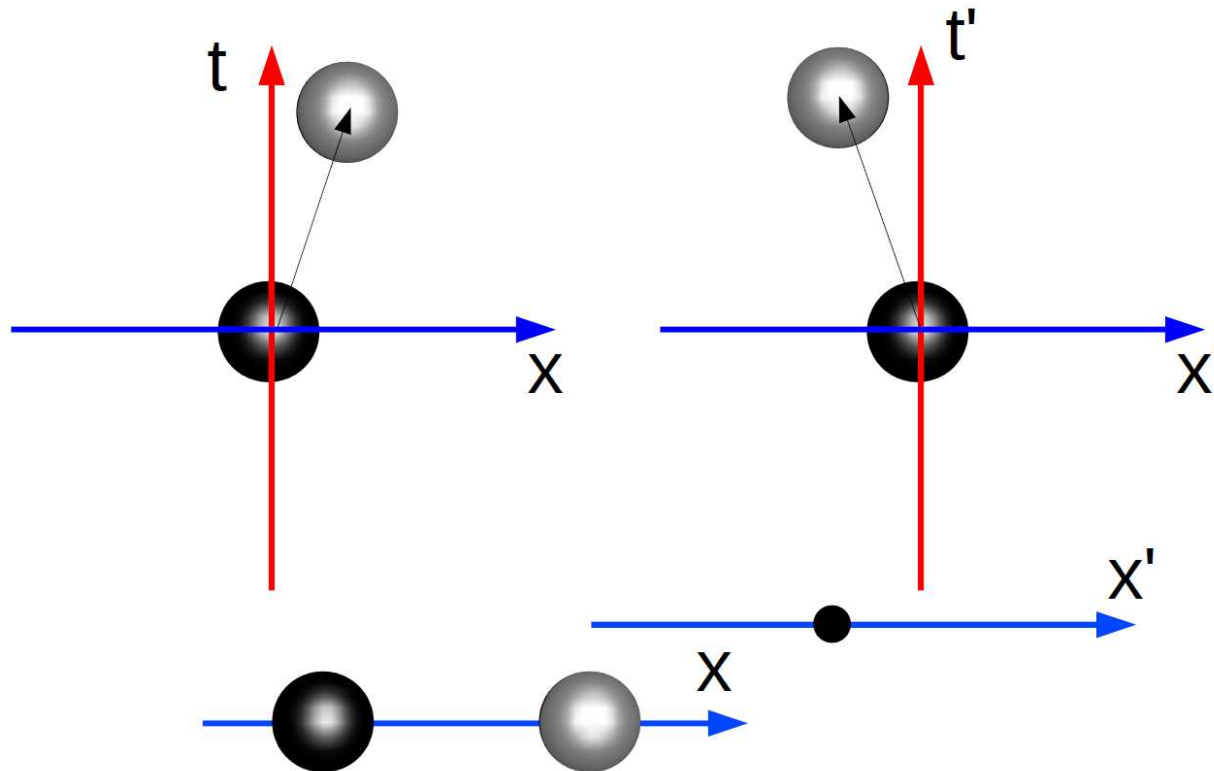
- Space-time: combination of **space** and **time**
  - 3 dimensions of **space** (though this is being questioned)
  - time** is the 4th dimension
- It is a mathematical model to write physics laws



# Space-Time

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- Space-time in classical physics (Galilean transformation)
  - ⇒ **time** is absolute and independent of **space**
- In Special Relativity (and later General)
  - ⇒ **time** and **space** are related ⇒ **time** different in frames



# Problem of Classical Physics (1900's)

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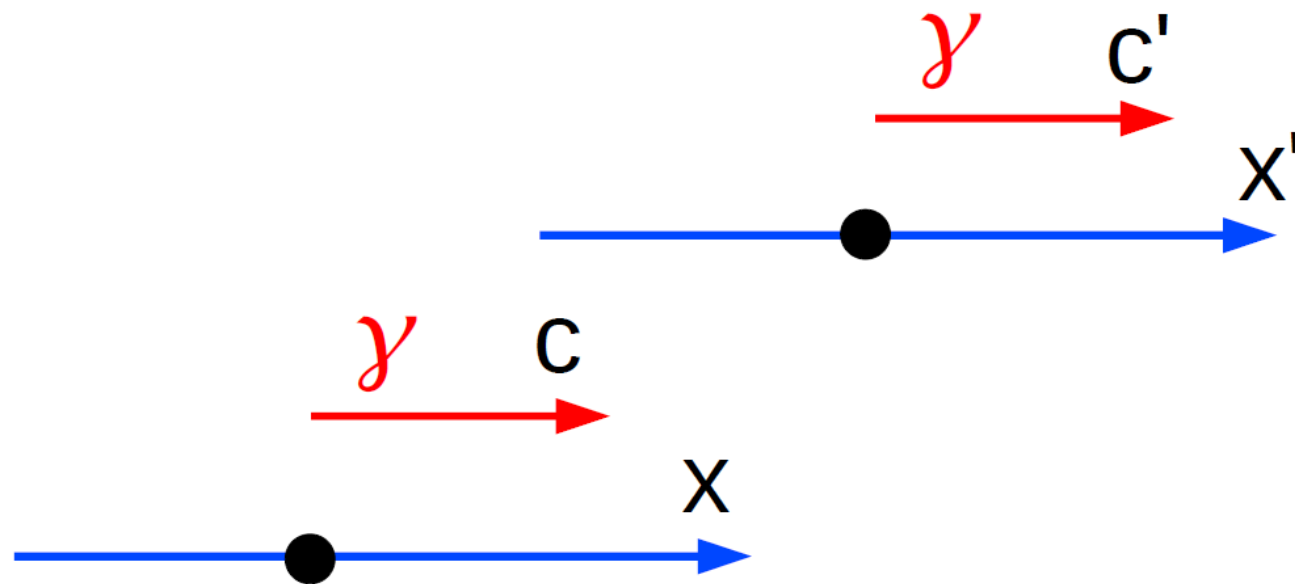
- Relativity and E&M looked inconsistent

relativity – same physics laws in all systems

E&M – Maxwell's equations involve speed of light =  $c$

Classically: speed of light different in different systems

classically  $c' = c - v$



# Postulate of Special Relativity

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- Einstein in 1905:
  - **relativity** valid – same physics laws in all systems
  - **speed of light** – same in all systems
- What is speed of light?  
 $c = 299\,792\,458 \text{ m/s} \simeq 3 \times 10^8 \text{ m/s}$
- Breaks classical Newtonian mechanics

$$t = t' \tag{1}$$

$$x = x' + v \times t' \tag{2}$$

$$y = y' \tag{3}$$

$$z = z' \tag{4}$$

# Consequence of Einstein's postulates

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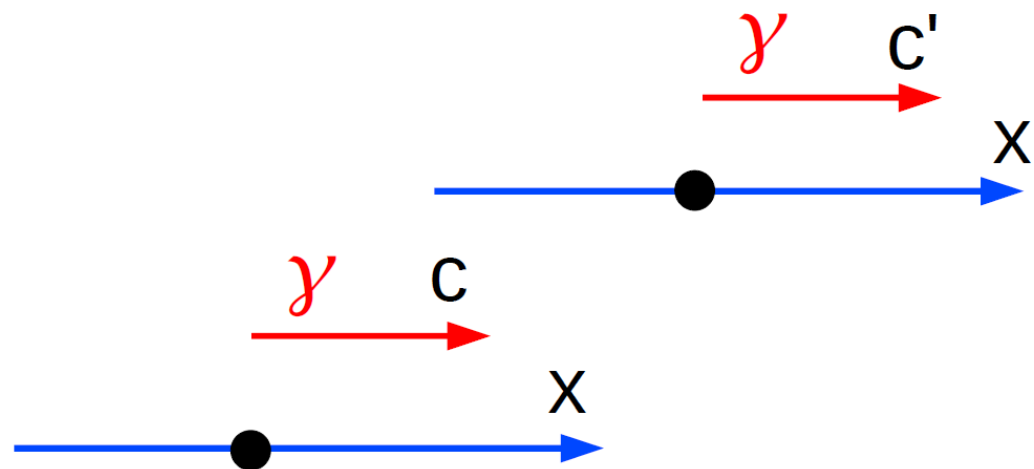
- Special Relativity

$$t = (t' + \frac{v}{c^2} \times x') / \sqrt{1 - \frac{v^2}{c^2}} \quad (5)$$

$$x = (x' + v \times t') / \sqrt{1 - \frac{v^2}{c^2}} \quad (6)$$

$$y = y' \quad (7)$$

$$z = z' \quad (8)$$



# Did the trick work?

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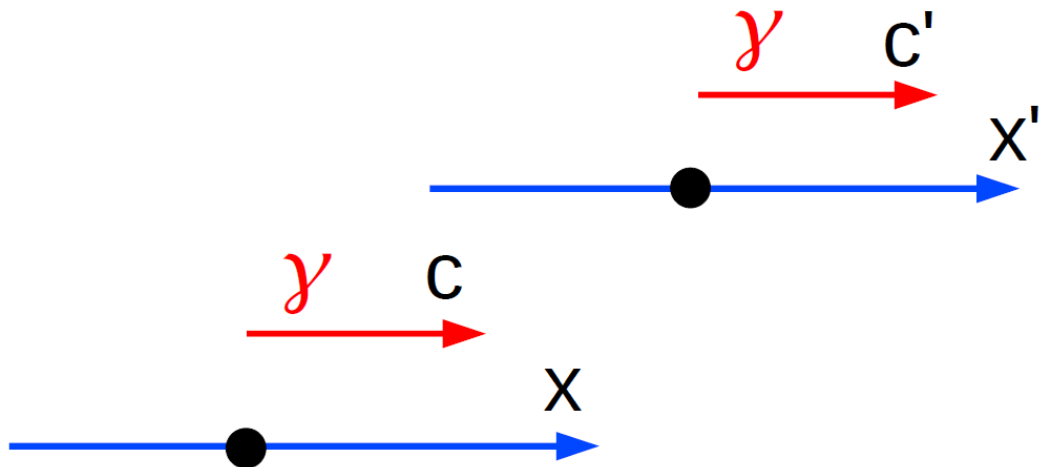
- A photon (light) will travel  $x = c \times t$

$$x = (x' + v \times t') / \sqrt{1 - \frac{v^2}{c^2}} = c \times t$$

$$t = (t' + \frac{v}{c^2} \times x') / \sqrt{1 - \frac{v^2}{c^2}}$$

$$\Rightarrow (x' + v \times t') = (ct' + \frac{v}{c} \times x')$$

$$\Rightarrow c' = \frac{x'}{t'} = \frac{c-v}{1-v/c} \equiv c \Rightarrow \text{same speed of light}$$



# Consequences

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- Length contraction

When a body moves with speed  $v$  relative to the observer, its length is contracted in the direction of motion

by  $\times \sqrt{1 - \frac{v^2}{c^2}}$

- Time dilation

When a clock moves with speed  $v$  relative to the observer, its rate is measured to have slowed down

by  $\times \sqrt{1 - \frac{v^2}{c^2}}$



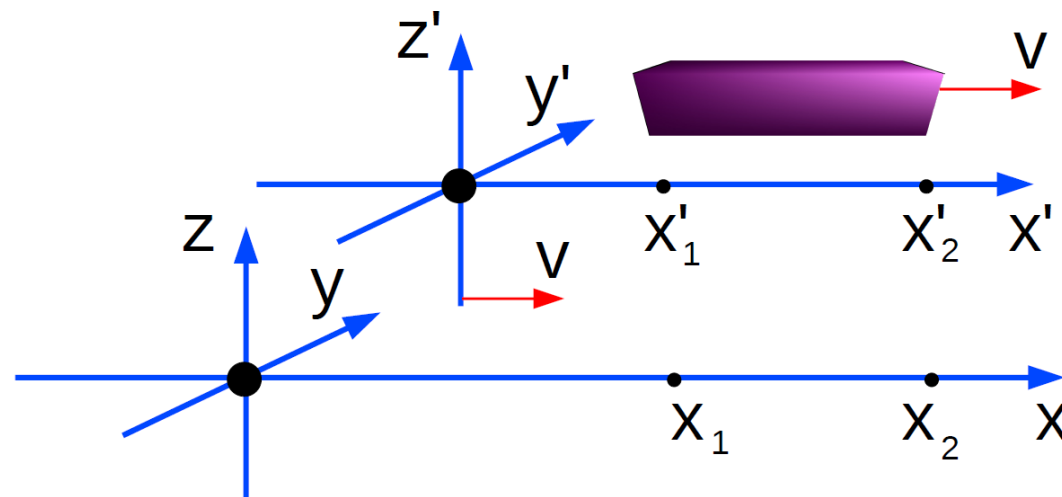
# Length contraction

- Consider a moving rod of length  $L$  in our frame

$$L = (x_2 - x_1) \text{ at } t_1 = t_2$$

- Length in its own frame  $L'_0 = (x'_2 - x'_1) > L$ , proof:

$$\begin{aligned} L'_0 &= (x_2 - v \times t_2) / \sqrt{1 - \frac{v^2}{c^2}} - (x_1 - v \times t_1) / \sqrt{1 - \frac{v^2}{c^2}} \\ &= L / \sqrt{1 - \frac{v^2}{c^2}} > L \Rightarrow L = L'_0 \times \sqrt{1 - \frac{v^2}{c^2}} \end{aligned}$$



# Time dilation

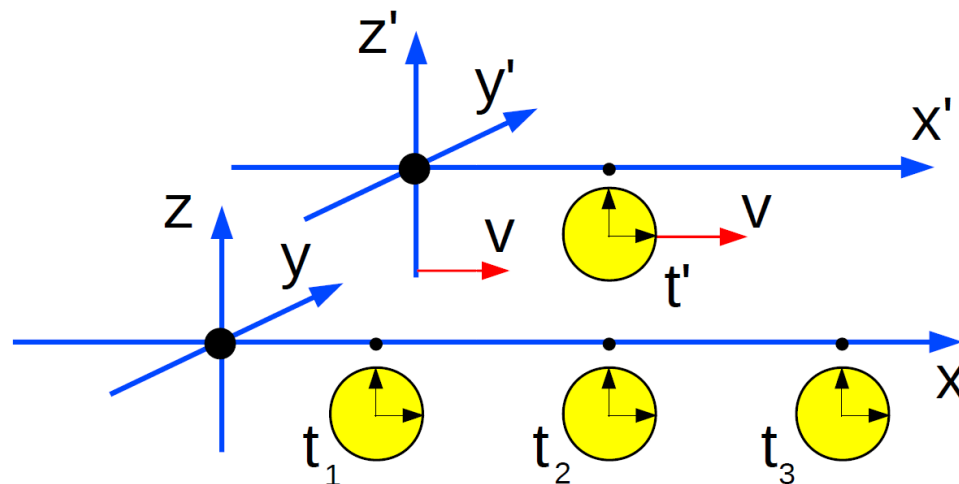
- Consider moving clock at  $x' = \text{const}$  in its own frame

$$\Delta t'_0 = (t'_2 - t'_1)$$

- Time between two events in our frame  $\Delta t = (t_2 - t_1)$

$$\Delta t = (t'_2 + \frac{v}{c^2}x') / \sqrt{1 - \frac{v^2}{c^2}} - (t'_1 + \frac{v}{c^2}x') / \sqrt{1 - \frac{v^2}{c^2}}$$

$$\Delta t = \Delta t'_0 / \sqrt{1 - \frac{v^2}{c^2}}$$



# Length contraction and time dilation

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- Length contraction

When a body moves with speed  $v$  relative to the observer, its length is contracted in the direction of motion

by  $\times \sqrt{1 - \frac{v^2}{c^2}}$

- Time dilation

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by  $\times \sqrt{1 - \frac{v^2}{c^2}}$

# Time dilation in particle physics

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- Example: lets take cosmic ray muon  $\mu^+$   
some facts: every minute 1 muon goes through 1 cm<sup>2</sup> area  
Quarknet: cosmic ray detectors, muon lifetime experiment
- Lets take typical muon energy 3 GeV
  - speed =  $0.9994 \times c$
  - lifetime  $\Delta t'_0 = 2.2 \times 10^{-6}$  seconds
  - naive distance traveled (if there were no time dilation)  
 $= 0.9994 \times c \times \Delta t'_0 = 659 \text{ m}$
  - but  $1/\sqrt{1 - \frac{v^2}{c^2}} = 29$
  - distance travel  $\Delta L = 659\text{m} \times 29 = 19\,000 \text{ m} = 19 \text{ km}$

# Time dilation: muon

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- Muon with  $v = 0.9994 \times c$  and  $\gamma = 1/\sqrt{1 - \frac{v^2}{c^2}} = 29$

- From our point of view

muon is moving speed  $v \simeq c$

time dilated

$$\Delta t = \Delta t'_0 \times \gamma$$

– distance  $\Delta L = 19 \text{ km}$

– time  $\Delta t = 64 \times 10^{-6} \text{ s}$

- From muon point of view

Earth is moving speed  $v \simeq c$

distance contracted

$$\Delta L'_0 = \Delta L / \gamma$$

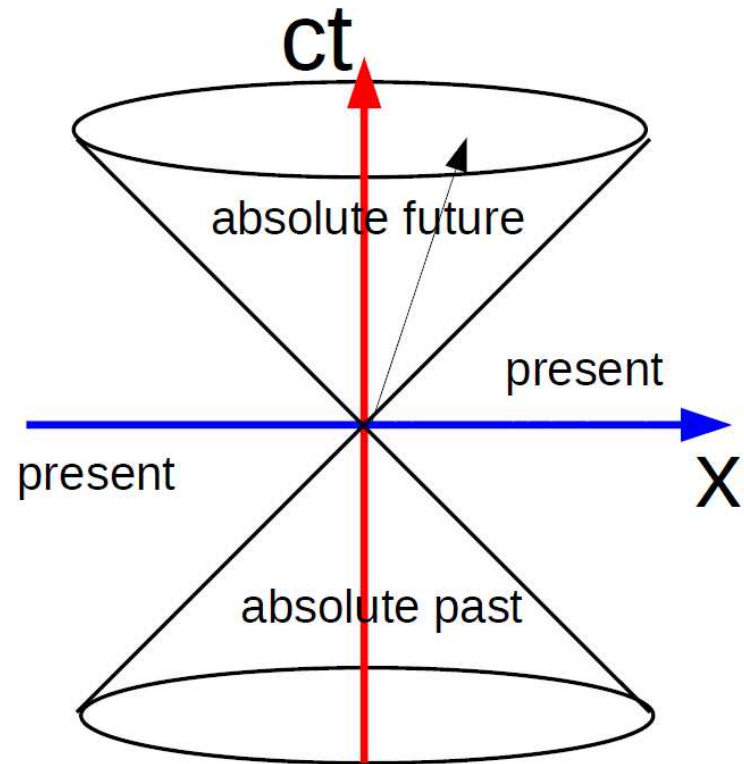
$$\Delta L'_0 = 659 \text{ m}$$

$$\Delta t'_0 = 2.2 \times 10^{-6} \text{ s}$$

# “Time Travel”

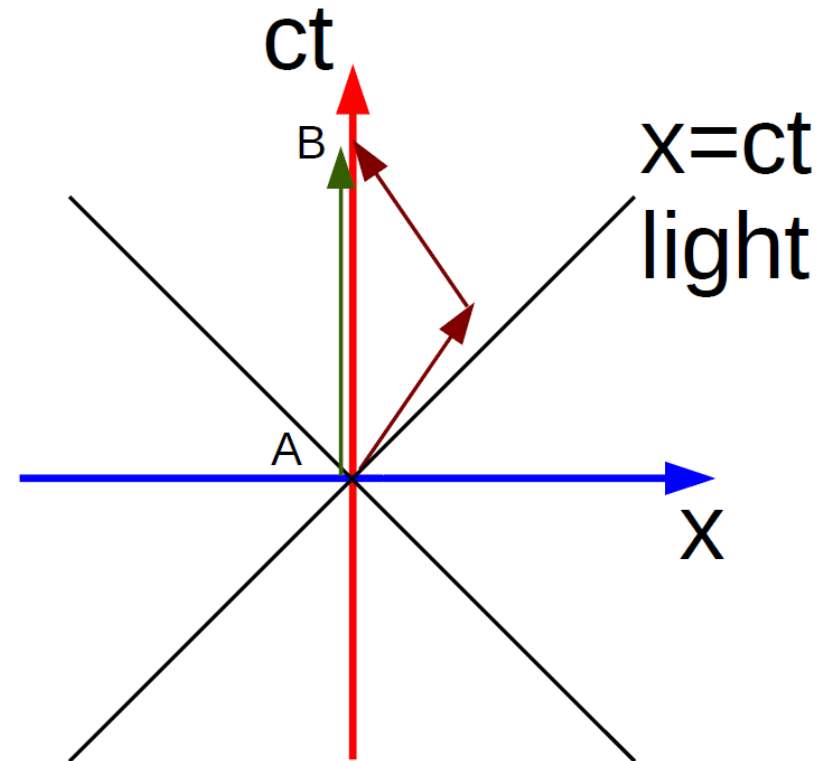
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- Cannot move “backwards in time”
- Move “forward” with different speed
- You can change “your clock”, but not direction:
  - biology: hibernate
  - physics: relativistic speed



# Twin paradox

- If one could reach  $v \simeq 0.9995 \times c$  (we will see why not)
  - time  $\gamma = 1/\sqrt{1 - \frac{v^2}{c^2}} \simeq 30$  times slower (like muon earlier)
  - get on a rocket and come back in 90 years
  - twin on Earth 90 years older
  - twin on rocket 3 years older



# Conservation of Energy and Momentum?

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- Energy and Momentum conserve, but different definition:

$$E = mc^2 = m_0c^2 / \sqrt{1 - \frac{v^2}{c^2}}$$

$$p = mv = m_0v / \sqrt{1 - \frac{v^2}{c^2}}$$

- Impossible to reach speed of light  $v = c \Leftrightarrow E = \infty$   
unless you are massless ( $m_0 = 0$ , like photon)
- Reverse is true: massless  $\Rightarrow v = c$  in all frames  
otherwise  $p = 0$  and  $E = 0 \Rightarrow$  like nothing
- One can convert mass into energy and energy into mass  
relativistic mass is energy